Data Fusion of the Quaternion and Non Linear Attitude Observer Applied to the Determination and Stabilization of a Mobile Robot

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Abstract. Generally, the attitude estimation and the measurement of the angular velocity are a requirement for the attitude control. As a result, the computational cost and the complexity of the control loop are relatively high. In the present paper, a technique for attitude stabilization is proposed. The rigid body orientation is modeled with quaternion, which eliminates attitude estimation singularities. The real-time implementation is done unifying a quaternion error formulation of Wahba's and a nonlinear observer. It includes the gyro bias model. A quaternion measurement model is introduced. It avoids the linearization step that induces undesirable effects. The global convergence of the proposed technique is proved. Simulations with some robustness tests are performed.

Keywords: Quaternion, Attitude, Stabilization, Nonlinear Observer, Robot Mobile, MEM's

1 Introduction

Underactuated mechanical systems are systems that have fewer control inputs than configuration variables. Underactuated systems appear in a broad range of applications including Robotics, Aerospace Systems, Marine Systems, Flexible Systems, Mobile Systems, and Locomotive Systems. The "underactuation" property of under actuated systems is due to the following four reasons [1]: *i) dynamics of the system* (e.g. aircraft, spacecraft, helicopters, underwater vehicles, locomotive systems without wheels), *ii) by design for reduction of the cost or some practical purposes* (e.g. satellites with two thrusters and flexible-link robots), *iii) actuator failure* (e.g. in a surface vessel or aircraft), *iv) imposed artificially to create complex low-order nonlinear systems* for the purpose of gaining insight in control of high-order underactuated systems (e.g. the Acrobot, the Pendubot, the Beam-and-Ball system, the Cart-Pole system, the Rotating Pendulum, the TORA system).

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A fundamental requirement for an autonomous vehicle is its ability to localize itself with respect to its environment. Navigation on a flat and horizontal ground only requires estimations of position and heading. However, in many cases, the environment is not so well structured, and the angular orientation of the vehicle may change along its path. In this case, a real time estimation of the attitude may be necessary.

The attitude estimation of an autonomous vehicle is a subject that has attracted a strong interest the last years. In addition to traditional attitude estimation in aerospace and automobile communities, the reduced cost of MEMS inertial sensors has spurred new applications in robotics [1], virtual reality [3] and biomechanics [2]. Furthermore, the increasingly interest in Unmanned Aerial Vehicles (UAVs) [4] has motivated the development of low cost, lightweight and low-power consumption Attitude and Heading Reference Systems (AHRS) and backup attitude indicators. An AHRS is composed of inertial and magnetic sensors, namely, three rate gyros, three accelerometer and three magnetometers, orthogonally mounted such that the sensor frame axes coincide with the body frame in question. In fact, an AHRS is an attitude estimator since the signal sensors are coupled with a proper mathematical background. This attitude estimation problem is described as following: Rate gyros provide continuous attitude information with good short-term stability when their measurements are integrated.

The first one deals with a constraint least-square minimization problem proposed firstly by Wahba [6],(see [7] pages 9-11) and [7].

The second approach is within the framework of the Extended Kalman Filter [8] (EKF) or Additive Extended Kalman Filter (AEKF) [9].

The third approach issues from nonlinear theory, and non linear observers are applied to the attitude determination problem [10], [12], [13]. In this approach, the convergence of the error to zero is proved in a Lyapunov sense.

In this paper, an attitude estimator using quaternion representation is studied. Here a novel method for solving Wahba's problem is used. This method allows to find an quaternion from the measures provided by an Attitude and Heading Reference Systems (AHRS), the error between current and desired orientations is directly determined thanks to the measurements of the reference vectors delivered by the body's sensors.

The present paper is organized as follows. In section 2 a quaternion-based formulation of the orientation of rigid body is given. The problem statement is formulated in section 3. The control law and attitude's estimation is presented in section 6. Simulation results are given in section 7. The paper ends with some concluding remarks given in section 8.

2 Mathematical Background

The attitude of a rigid body can be represented by a unit quaternion, consisting of a unit vector e, known as the Euler axis, and a rotation angle β about this axis. The quaternion q is then defined as follows:

$$q = \begin{pmatrix} \cos\frac{\beta}{2} \\ e\sin\frac{\beta}{2} \end{pmatrix} = \begin{pmatrix} q_0 \\ q \end{pmatrix} \in H \tag{1}$$

where

$$H = \{q \mid q_0^2 + q^T q = 1, \\ q = [q_0 q^T]^T, q_0 \in R, \ q \in R^3\} \tag{2}$$

$q = [q_1 \ \ q_2 \ \ q_3]^T$ and q_0 are known as the vector and scalar parts of the quaternion respectively. In attitude control applications, the unit quaternion represents the rotation from an inertial coordinate system $N(x_n, y_n, z_n)$ located at some point in the space (for instance, the earth NED frame (North, East, Down)), to the body coordinate system $B(x_b, y_b, z_b)$ located on the center of mass of a rigid body.

If r is a vector expressed in N, then its coordinates in B are expressed by:

$$b = \bar{q} \otimes r \otimes q \tag{3}$$

where $b = [0 \ \ b^T]^T$ and $r = [0 \ \ r^T]^T$ are the quaternions associated to vectors b and r respectively. $\otimes$ denotes the quaternion multiplication and $\bar{q}$ is the conjugate quaternion of q, defined as:

$$\bar{q} = [\, q_0 \ -q^T \,]^T \tag{4}$$

The rotation matrix $C(q)$ corresponding to the attitude quaternion q, is computed as:

$$C(q) = (q_0^2 - q^T q)I_3 + 2(qq^T - q_0[q^\times]) \tag{5}$$

where I_3 is the identity matrix and $[\xi^\times]$ is a skew symmetric tensor associated with the axial vector ξ:

$$[\xi^\times] = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}^\times = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix} \tag{6}$$

Thus, the coordinate of vector r expressed in the B frame is given by:

$$b = C(q)r \tag{7}$$

The attitude error is used to quantify the mismatch between two attitudes.

$$q_e = q \otimes q_d^{-1} \tag{8}$$

$\otimes$ denotes the quaternion multiplication and q_d^{-1} is the complementary rotation of the quaternion q_d, which is the quaternion conjugate (see ([5]) for more details).
The attitude dynamics of a rigid body is described by

$$J\dot{\omega} = -\omega \times J\omega + \Gamma \tag{9}$$

where $J \in R^{3\times 3}$ is the symmetric positive definite constant inertial matrix of the rigid body expressed in the B frame and $\Gamma \in R^3$ is the vector of control torques. Note that the torque also depend on the environmental disturbance (aerodynamic, gravity gradient, etc.).

3 Problem Statement

In the case of the attitude estimation, one seeks to estimate the attitude and accelerations of a rigid body. From now on, it is assumed that the system is equipped with a tri-axis accelerometer, three magnetometer and three rate gyros mounted orthogonally. In this section we describe the body's kinematic of the model [15], our typical capture configuration relies primarily on the Robot of figure 1 equipped with two 6 inch diameter wheels driven by 1 DC gear head motors. The mechanical model (seen in figure 1, is based on single pinion architecture suitable for light vehicles and consists of following elements: a steering rack, a steering column coupled to the steering rack through a pinion gear, and the assist motor. Tie-rods connect the steering rack to the tires.

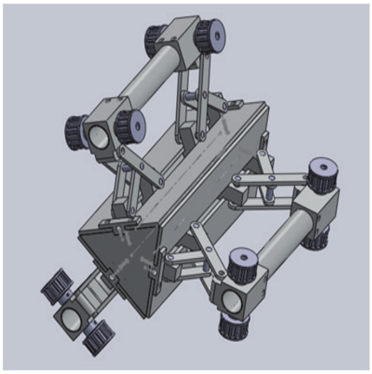


Fig. 1. Mobile Robot

The equation describing the relation between the quaternion and the body's kinematic is given in introducing the angular variation $\omega = [\omega_x \ \omega_y \ \omega_z]^T$ from this, it follows.

$$\dot{q} = \frac{1}{2}\Omega(\omega)q(t) = \frac{1}{2}\Xi(q)\omega(t) \tag{10}$$

Where $\Omega(\omega)$ y $\Xi(q)$ are defined as:

$$\Omega(\omega) \equiv \begin{bmatrix} -[\omega\times] & \vdots & \omega \\ \dots & \vdots & \dots \\ -\omega^T & \vdots & 0 \end{bmatrix} \tag{11}$$

$$\Xi(q) \equiv \begin{bmatrix} q_0 I_{3\times3} + [q\times] \\ \dots\dots\dots\dots \\ -q^T \end{bmatrix} \tag{12}$$

The matrix $[\omega\times]$ and $[q\times]$ are obtained by the cross product issue of $a\times b=[a\times]b$ con $[a\times]\in R^{3\times 3}$:

$$[a\times]=\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{13}$$

The quaternion must be:

$$q^T q = q^T q + q_0^2 = 1 \tag{14}$$

In the other hand, the matrix $\Xi(q)$ has the relation:

$$\begin{array}{l} \Xi^T(q)\Xi(q) = q^T q I_{3\times 3} \\ \Xi^(q)\Xi^T(q) = q^T q I_{4\times 4} - q^T q \\ \Xi^T(q)(q) = 0_{3\times 1} \end{array} \tag{15}$$

Generally $\Xi^T(q)\lambda = -\Xi^T(\lambda)q$, for any $\lambda \in H$.

$$\begin{array}{l} C(q) = (q_0^2 - q^T q)I_{3\times 3} + 2qq^T \\ \quad -2q_0[q\times] \end{array} \tag{16}$$

that is denoted like the orientation matrix 3-D of dimension 3×3.

The angular velocity ω is obtained by finite differences from equation (10) at the instants k and $k-1$ (k estimation instant).

$$\omega = 2\Xi^T(q)\dot{q} \tag{17}$$

$$\omega = 2\Xi^T(q) * \left(\frac{q(k)-q(k-1)}{Ts}\right) \tag{18}$$

4 Modeling sensors

In application of inertial and magnetic sensors, the inertial coordinate frame N is chosen to be the NED coordinate frame. In this work the origin of N is located at San Luis Potosí, México $(GPS : 22^\circ 36' 12'' N\ 100^\circ 25'\ 47'' W)$. The "reference vectors" are the gravitational and magnetic vectors, which are well known. The "vectors observations", i.e. the gravitational and magnetic vectors expressed in the body frame B, are obtained from an tri-axis accelerometer and a tri-axis magnetometer sensors. The angular velocity is obtained from three rate gyros orthogonally mounted.

4.1 Rate Gyros

The angular velocity $\omega = [\omega_1\ \omega_2\ \omega_3]^T$ is measured by the rate gyros, which are supposed to be orthogonally mounted. The output signal of a rate gyro is influenced by various factors, such as bias drift and noise. In the absence of rotation, the output signal can be modeled as the sum of a white Gaussian noise and of a slowly varying function, an integration process is required in order to obtain the current attitude quaternion.

The kinematics equation is given by:

$$\begin{aligned}\begin{pmatrix}\dot{q}_0\\ \dot{q}\end{pmatrix} &= \frac{1}{2}\begin{pmatrix}-q^T\\ I_3 q_0 + [q^\times]\end{pmatrix}\omega \\ &= \frac{1}{2}\Xi(q)\omega\end{aligned} \tag{19}$$

Even the smallest variation of the rate gyro measurement will produce a wrong estimation of the attitude . The bias is denoted by ν, belonging to space R^3. The rate gyro measurements are modeled by [14]:

$$\omega_G = \omega + \nu + \eta_G \tag{20}$$

$$\dot{\nu} = -T^{-1}\nu + \eta_\nu \tag{21}$$

where η_G and $\eta_\nu \in R^3$ are supposed by Gaussian white noises and $T = \tau I_3$ is a diagonal matrix of time constants. In this case, the constant τ which has been set to $\tau = 100\ s$. The bias vector ν will be estimated online, using the observer presented in the following section.

4.2 Accelerometers

Since the $3-axis$ accelerometer is fixed to the body, the measurements are expressed in the body frame B. Thus, the accelerometer output can be written as:

$$b_A = C(q)(a-g) + \eta_A \tag{22}$$

where $g = [0\ \ 0\ \ g]^T$ and $a \in R^3$ are the gravity vector and the inertial accelerations of the body respectively. Both are expressed in frame N. $g = 9.81\ m/sec^2$ denotes the gravitational constant and $\eta_A \in R^3$ is the vector of noises that are supposed to be white Gaussian.

4.3 Magnetometers

The magnetic field vector h_M is expressed in the N frame it is supposed to be $h_M = [h_{M_x}\ \ 0\ \ h_{M_z}]^T$. Since the measurements take place in the body frame B, they are given by:

$$b_M = C(q)h_M + \eta_M \tag{23}$$

where $\eta_M \in R^3$, denotes the perturbing magnetic field. This perturbation vector is supposed to be modeled by Gaussian white noises.

5 Attitude's Estimation and Prediction

The attitude estimator uses quaternion representation. Two approaches are jointly used, namely a estimation with a constraint least-square minimization technique and a prediction of the estate at the instant k. The prediction is performed in order to produce a pseudo-estimate of the accelerations and the attitude quaternion. This prediction is driven by a estate which is obtained from the quaternion propagated through the kinematic equation and the one obtained via the constraint minimization problem.

In this paper a optimization criteria that take in account the evolution of the attitude state via determination of $x = [q_0, q_1, q_2, q_3, a_x, a_y, a_z]^T$ in the function $f(x)$ is proposed. The minimum error is chosen, but it takes in account the prediction of the state $\hat{x}$ and the coefficients of weight for the estate μ and the measures estimated ($MesEstimated = MS$) at the instant k.

$$\begin{aligned} f(x) = \tfrac{1}{2}[\mu(\textstyle\sum_{j=1}^{n}(MesEstimated - v_{mes}(j))^2) \\ +(1-\mu) \parallel (\hat{x}-x) \parallel_2^2] \end{aligned} \tag{24}$$

$$with \qquad q^T q - 1 = 0$$

The process of Estimation and Prediction needs the determination of their gradient g_q, g_a and their Hessian H_q, H_a.

$$H_q = \left[\frac{\partial^2 f}{\partial q^2} \cdot \frac{\partial q}{\partial x}\right]^T \tag{25}$$

$$\frac{\partial g_q}{\partial a} = \begin{bmatrix} 2\sum_{j=1}^{3}\left(2\left(q^T MS_j q - v_{mes}(j)\right)\frac{\partial MS_j}{\partial MS_1}q + q^T\frac{\partial MS_j}{\partial MS_1}qMS_jq\right) \\ 2\sum_{j=1}^{3}\left(2\left(q^T MS_j q - v_{mes}(j)\right)\frac{\partial MS_j}{\partial MS_2}q + q^T\frac{\partial MS_j}{\partial MS_2}qMS_jq\right) \end{bmatrix}. \tag{26}$$

Similarly, is the obtention for the gradient of the state for the case of acceleration.

Finally, the total Gradient is obtained by the fusion between the calcule show for the quaternion case an the gradient omitted for the acceleration case.

$$F(x) = \begin{bmatrix} H_q & \frac{\partial g_q}{\partial a} \\ \frac{\partial g_a}{\partial x} & H_a \end{bmatrix} \tag{27}$$

For the prediction's process of $\hat{x}$, several technique have been validated, for purpose of simplicity, the prediction via spline is chosen. Cubic spline is a spline constructed of piecewise third-order polynomials which pass through a set of n control points. The second derivative of each polynomial is commonly set to zero at the endpoints, since this provides a boundary condition that completes the system of $n-2$ equations. This produces a so-called "natural" cubic spline and leads to a simple tridiagonal system which can be solved easily to give the coefficients of the polynomials. However, this choice is not the only one possible, and other boundary conditions can be used instead.

Suppose we are given $n+1$ data points $(\hat{x}_k, MS_k)$ such that.

$a = x_0 < \ldots < x_n$, Then the coefficients of the vector μ exists cubic polynomials with coefficients $\mu_{j,i} \qquad 0 \leq i \leq 3$ such that the following hold.

1. $\mu(\hat{x}) = \mu_j(\hat{x}) = \sum_{j=0}^{3}(\hat{x}-x_j)^i \forall \hat{x} \in [\hat{x}-x_{j+1}]\, 0 \leq k \leq n-1$
2. $\mu(x_j) = y_k \qquad 0 \leq k \leq n-1$
3. $\mu_j(x_{j+1}) = \mu_{j+1}(x_{j+1}) \qquad 0 \leq k \leq n-2$

4. $\mu_j'(x_{j+1}) = \mu_{j+1}'(x_{j+1}) \qquad 0 \leq k \leq n-2$
5. $\mu_j''(x_{j+1}) = \mu_{j+1}''(x_{j+1}) \qquad 0 \leq k \leq n-2$

So we see that the cubic spline not only interpolates the data $(\hat{x}_k, MS_k)$ but matches the first and second derivatives at the knots. Notice, from the above definition, one is free to specify constraints on the endpoints. The end point constraint $\mu''(a) = 0 \quad \mu''(b) = 0$ is chosen.

The estimation of the torque is part of another work that is in process and only we present his basic model. Since the driver torque is not measured, we introduce another estimator for $\Gamma_{Mot} = \Gamma$,

Essentially, the estimated value of the driver torque is

$$\Gamma_{iest} = G^{(-1)}(\Gamma_{LZ}(z) - H(z).\Gamma_{Mot(z)}) \tag{28}$$

Where Γ_{LZ} is the torque in the steering column part and Γ_{Mot} is the assist motor torque. In order that $G^{(-1)}$ can be physically realizable (numerator degree of the transfer function is always less or equal than denominator degree), it is necessary to introduce a correction transfer function $G_c(z)$ to maintain the properness. With this correction, the inverse transfer function becomes

$$\Gamma_{iest} = G^{(-1)} * G_c(z) * (\Gamma_{LZ}(z) - H(z).\Gamma_{Mot(z)}) \tag{29}$$

6 Nonlinear attitude observer

The attitude nonlinear observer that includes the bias and the error update is given by:

$$\dot{\hat{q}} = \frac{1}{2}\Xi(\hat{q})\left[\omega_G - \hat{\nu} + K_1\varepsilon\right] \tag{30}$$

$$\dot{\hat{\nu}} = -T^{-1}\hat{\nu} - K_2\,\varepsilon \tag{31}$$

where T has been defined in (21) and $K_i, i = 1,2$ are positive constant parameters. $\hat{q}$ is the prediction of the attitude at time t. It this obtained via the integration of the kinematics equation (30) using the measured angular velocity ω_G, the bias estimate $\hat{\nu}$ and $\varepsilon = q_e$ which is the vector part of the quaternion error q_e. Remember that q_e measures the discrepancy between $\hat{q}$ and the pseudo-measured attitude q_{ps} (32). In this paper, q_{ps} is obtained thanks to an appropriate treatment of the accelerometer and magnetometer measurements.

Combining (19),(21), (30) and (31) the error model is expressed as:

$$\dot{q}_e = \frac{1}{2}\begin{pmatrix} 0 & \gamma^T \\ -\gamma & [2\omega^\times] + [\gamma^\times] \end{pmatrix}\begin{pmatrix} q_{e_0} \\ q_e \end{pmatrix} \tag{32}$$

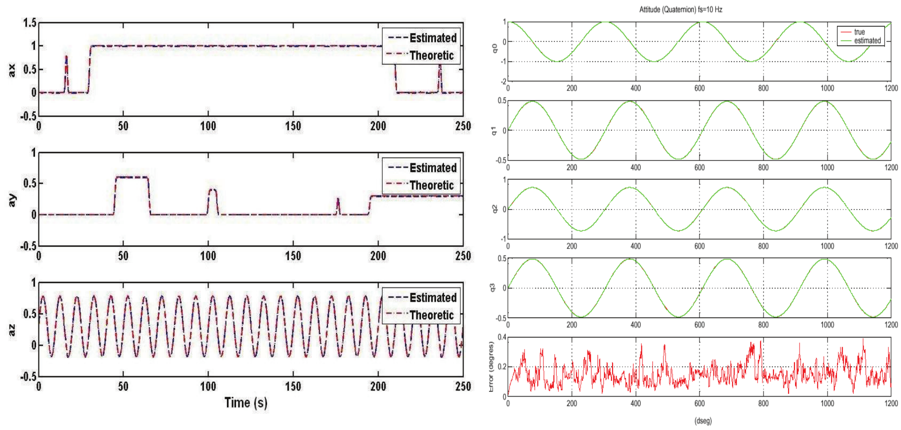
$$\dot{\tilde{\nu}} = -T^{-1}\tilde{\nu} + K_2\,\varepsilon \tag{33}$$

where $\gamma = \tilde{\nu} + K_1\varepsilon$, and $\tilde{\nu} = \nu - \hat{\nu}$. The system (33)-(32) admits two equilibrium points $(q_{e_0} = 1\ q_e = 0\ \tilde{\nu} = 0)$ and $(q_{e_0} = -1\ q_e = 0\ \tilde{\nu} = 0)$. This is due to fact that quaternions q and $-q$ represent the same attitude. From (1), one obtains:

$$q_{e_0} = 1 \quad \Rightarrow \beta = 0$$
$$q_{e_0} = -1 \ \Rightarrow \beta = 2\pi \ (generally\ 2n\pi)$$

7 Validation

In this section, some simulation results are presented in order to show the performance of the proposed control laws. A rigid body with low moment of inertia is taken as the experimental system. In fact, the low moment of inertia makes the system vulnerable to high angular accelerations which proves the importance to apply the control.



Estimation and Prediction of the Acceleration   Estimation and Prediction of the Quaternion

Fig. 2. Validation de Movements

The proposed technique is compared to the existing methods (namely, the Multiplicative Extended Kalman Filter (MEKF) and the Additive Kalman Filter (AEKF)). Initial conditions are set to extreme error values in order to assess the effectiveness of attitude estimation. These results are depicted in figures 2.

8 Conclusion and future works

In this paper, a control law for the global stabilization of a rigid body was proposed. The presented methodology is especially simple. It is based on quaternion error and a nonlinear observer the attitude is parameterized by the unit quaternion. Furthermore, the proposed approach can be extended to the stabilization of a pico-satellite or a micro-satellite. Remain to perform several validations in the robot mobil and to those provided by a vision-based human motion capture system that will be used as a reference attitude estimation system and embebed in robots to assist people and improve human performance in daily and task-related activities, focusing in particular on populations with special needs, including those convalescing from trauma, rehabilitating from cognitive and/or physical injury, aging in place or in managed care, and suffering from developmental or other social, cognitive, or physical disabilities. Another application desired for the presented approach is the stabilization of micro-satellite and UAV, simulations using the dimension and the actuator characteristics of a pico-satellite and a micro-satellite and to compare the proposed approach with other control schemes.

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